Application of Reliability Theory for Stopping Sight Distance & Radius to Highways Horizontal Curves Design in Egypt

[1]Alaa Ahmed Ali Ahmed, [2] Ibrahim Mohammed Ramada, [3]Ahmed Abd El Moamen Khalil, [4]Walid Ahmed Dawoud

[1]M.Sc in Civil Engineering , Faculty of Engineering at Shoubra –Banha University

[2]Associate Prof.Transportation &Highways Eng.-Civil Eng. Department Faculty of Engineering at Shoubra – Banha University

[3]Associate Prof.Railways & Transportation Eng.-Civil Eng. Department Faculty of Engineering at Shoubra – Banha University

[4]Assistant Prof .Geotechnical Eng.-Civil Eng. Department Faculty of Engineering at Shoubra –Banha University

تطبيق نظرية الموثوقية لمسافة الرؤيا ونصف القطر فى تصميم المنحيات الأفقية للطرق السريعة فى مصر

الملخص :

تهدف هذه الدراسة الي تطبيق تحليل الموثوقية لتقييم مجموعة من المنحنيات وعددها تسعة منحنيات علي الطرق الخلوية المتعددة الحارات الموجودة بالقاهرة الكبرى بمصر وذلك لتحديد مدي مناسبة مسافة الرؤية المتاحة علي تلك المنحنيات لمسافة الرؤية المطلوبة من مستخدمي الطريق وكذلك مدي مناسبة نصف القطر المتاح لنصف القطر المطلوب لمستخدمي الطريق.

بالنسبة لمسافة الرؤية فإن احتمالية تخطى حد الأمان تحدث عندما تتخطي مسافة الرؤية المطلوبة لقائدي المركبات علي كل منحني مسافة الرؤية المتاحة فعليا، حيث أن مدخلات حساب مسافة الرؤية المطلوبة متغيرات عشوائية ولها توزيع احتمالي وهي السرعة وزمن الادراك ورد الفعل والعجلة التناقصية .تم فرض متوسط زمن الادراك ورد الفعل وكذلك العجلة التناقصية للمركبة كما بالدراسات السابقة .بينما تم حساب القيم المتوسطة للسرعة التشغيلية من خلال تجميع بيانات السرعة اللحظية علي المنحنيات تحت الدراسة بخلاف الدراسات السابقة والتي استخدمت معادلات محدده لحساب متوسط السرعة التشغيلية علي كل منحني وكانت مسافة الرؤية المتاحة علي كل منحني محددة .أظهرت النتائج أن حوالي 78 % من المنحيات التي تمت دراستها لها إحتمالية تخطى حد الأمان أقل من10 %.

كما تم تقييم المنحنيات من ناحية نصف القطر حيث أن إحتمالية تخطى حد الأمان في التشغيلية للمنحني تحدث عندما تتخطي قيمة نصف القطر المطلوب من مستخدمي الطريق قيمة نصف القطر المتاح .وكانت مدخلات حساب نصف القطر المطلوب هي السرعة ومعامل الاحتكاك الجانبي للاطارات مع الاسفلت وإرتفاع الظهر عن البطن حيث ان كلا من السرعة والإحتكاك الجانبي متغيرات عشوائية بينما إرتفاع الظهر عن البطن قيمة محددة، بينما نصف القطر المتاح قيمة محددة . وأوضحت النتائج احتمالية تخطى حد الأمان يساوى صفر% بمعنى كل المنحنيات درجة الموثوقية بنسبة 100 %

Abstract: The main objective of this research is to evaluate the risk related to geometric design features for horizontal curves using reliability analysis. In this approach, Variables in the design equations are treated as random variables. Appropriate design of geometric elements in roads is an important feature for safety measures. The stopping sight distance (SSD) and the radius (R) of the horizontal curve are the most important geometric elements at the horizontal curve section. Therefore, the objective of this research is to check the probability of noncompliance of a given horizontal curve taking into consideration the random distribution of the various elements of a horizontal curve. Both SSD and R increase with the increase in speed. However, the available sight distance (ASD) at site can be regulated by varying R and middle ordinate (M) at the curve section. This study focuses on the intercity rural freeways roads in Egypt. Traffic and geometry data were collected for 9 curves from 3 freeway roads. All roads have posted speed limit of 100 to 120 km/h, Horizontal curves were selected to cover the different geometric characteristics like (middle ordinate, radius, super elevation, deflection angle, lane width and shoulder width) and traffic data like Spot speeds. Reliability analysis using Minitab program focuses on two modes of noncompliance in horizontal curve design. The first mode is insufficient sight distance which means that available sight distance (ASD) on the horizontal curve is less than required stopping sight distance (SSD). The second mode of noncompliance occurs when the required horizontal curve radius (R_{demand}) exceeds the available horizontal curve radius (R_{av}). The results showed that all curves satisfy the demand radius by users which have a reliability equal 100% , In the other hand probability of non-compliance is calculated to evaluate the stopping sight distance , where non-compliance occurs whenever the available sight distance (ASD; supply) falls short of the stopping sight distance (SSD; demand). Then, the contributions of uncertainty in the geometric variables were evaluated. The results showed that about 78% of studied curves have reliability exceed 90%.

Keywords: Reliability- Horizontal curves- Highway safety

1. Introduction

This study evaluates 9 horizontal curves in greater Cairo, Egypt, the available curve radius of the existing curves are compared with the required curve radii, where noncompliance occurs whenever the required curve radius exceeds the available curve radii. The inputs of the required radius are the operating speed, the lateral friction and the superelevation. Both of the operating speed and the lateral friction are random variables and the superelevation is deterministic and its value is determined from data collection. The mean of the lateral friction is obtained based on the previous model and the standard deviation is assumed as the previous studies. However the mean and the standard deviation of the operating speed are obtained based on spot speed data collected at sites. On another hand the available curve radius is deterministic. In addition to this study evaluates the available sight distance (ASD; supply) comparing to the stopping sight distance (SSD; required). Where noncompliance occurs whenever the SSD exceeds the ASD. The inputs of the SSD are the operating speed, driver’s perception reaction time (PRT) and deceleration of vehicles (a). All of them are random variables. Based on the previous studies the mean value and the standard deviation are assumed for "PRT" and "a". However the operating speeds, this study provides a new method to calculate the mean and standard deviation based on spot speed data collected at sites, instead of using the previous models for calculation, as the case in the previous work. The inputs of ASD are deterministic.

2. Literature Review

Most studies used to evaluate the risk related to geometric design features for horizontal curves using reliability analysis, A good knowledge of the reliability of the individual elements is essential to design a “consistent” highway was stated by *Felipe (1996).*. The work investigated the probabilities of rollover and sideslip for the minimum radius guided by

AASHTO Green Book, using FORM and limit state functions, respectively. *Hussein et al. (2014)* provide calibrated design charts for the middle ordinate M defined as the lateral distance between edge of median barriers and centerline of the adjacent traffic lane, at different probability of noncompliance levels. The results show that the calibrated values of M are generally lower than those derived from the AASHTO design guide. *Dhahir and Hassan (2015)* through implementing a reliability based probabilistic approach. In this study the distribution of available lateral friction under wet pavement conditions is taken based on the results of previous research. A new model was developed by authors based on the results of the output of vehicle dynamics simulation software to determine the distribution of demand lateral friction. First Order Reliability Method (FORM) is used to determine the reliability index and probability of failure (POF). *Easa et al. (2016)* demonstrates the application of multi-mode reliability analysis to the design of horizontal curves. The process is demonstrated by a case study of Sea-to-Sky Highway located between Vancouver and Whistler, in southern British Columbia, Canada. *Fambro et al. (2000)* evaluated the new SSD model against the old model that was presented in AASHTO (1994) and other models in the literature. *Faghri and Demetsky (1988)* and *Easa (1994)* demonstrated the potential of using reliability to evaluate limitations in sight distance at road-railway grade crossings. *Navin (1990)* carried out a study to investigate whether safety measures for stopping sight distance, horizontal curves, decision sight distance and passing sight distance could be developed. A failure of a specific approach as the case when its ASD does not meet its RSD was defined by *Easa (2000).* Another term, referred to as a system failure, was also used to indicate failures in two approaches forming a sight triangle at a specific intersection. The effect that narrow medians have on horizontal curves with restricted sight distance was investigated by *Richl and Sayed (2006).* The authors applied first-order reliability analysis for studying available sight distance at locations with median barriers on horizontal curves. Monte-Carlo simulation was used by *Sarhan and Hassan (2008)* to compute the probability of non-compliance associated with insufficiency of sight distances. Due to the lack of data, the authors used a computer program to develop design parameters to calculate the profiles of sight distances in two and three dimensional projections. *Ibrahim (2011)* tried to incorporate a reliability-based quantitative risk measure in the development of Safety Performance Functions (SPFs). The author considered the design of horizontal curves, where non-compliance occurs whenever the available sight distance (ASD; supply) falls short of the stopping sight distance (SSD; demand). Different methodologies of modeling the ASD were compared by *Santos-Berbel et al. (2017).* The authors analyzed The ASD of 402 horizontal curves, located in twelve in-service two-lane rural highways. Three ASD estimation methods were used which include a 2D method and two different 3D methods. The ASD results obtained through 2D and 3D methodologies are compared. Also, the different conditions of the existing road side features or geometric elements, under which the 3D ASD estimation is important, were identified. Next, reliability theory is utilized to evaluate the risk level (Pnc) associated with limited sight distance for each ASD modeling method. Stopping sight distance in horizontal curves was evaluated by *Rajbongshi and Kalita (2018).* Authors present a probabilistic approach for evaluating stopping sight distance, considering the variability of all input parameters of sight distance. *Osama El shotairy,Ibrahim Hashim and Ahmed Abu El-maaty (2019)* presented evaluation of 12 horizontal curves for two way two lane highway and The results showed that about 85% of studied curves have probability of non-compliance (Pnc) less than 10% for ASD and The results showed that about 85% of studied curves have probability of non-compliance (Pnc) less than 10%.

3. Performance Function uses for Reliability

This study focuses on two modes of noncompliance in horizontal curve design. The first mode is insufficient sight distance which means that available sight distance (ASD) on the horizontal curve is less than required stopping sight distance (SSD). The second mode of noncompliance occurs when the required horizontal curve radius ($Rdemand$) exceeds the available horizontal curve radius (Rav). Figure (1) shown that.

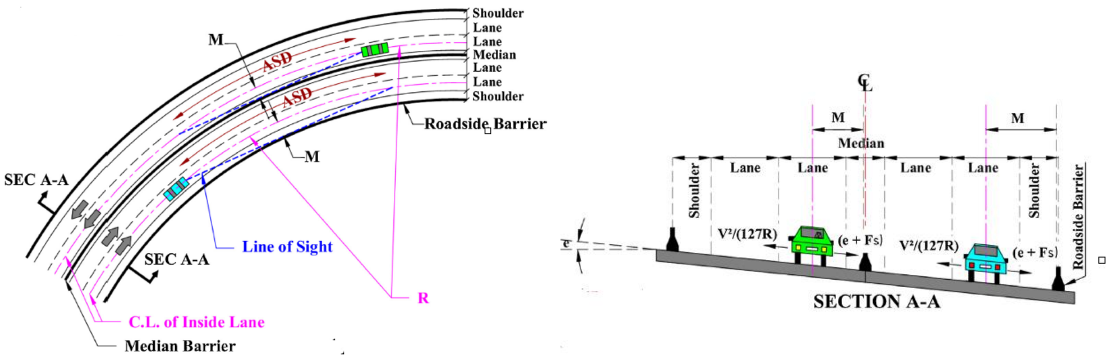

Fig.(1): Horizontal curve design elements used in the analysis

3.1 the first mode according to sight distance

The limited state funtion(G1) to evaluate the sight distance of the horizontal curves

$$G1 = ASD - SSD \quad (1)$$

Where:
ASD = the Available Sight Distance,and
SSD = the Stopping Sight Distance.
And non-compliance occurs when ASD is less than SSD (g < 0).
ASD is the portion of the road currently available to the driver. For horizontal curves, the ASD (i.e., the available variable) is calculated as Equation 3.2, AASHTO (2011).

$$ASD = 2R\cos^{-1}(1 - \frac{M}{R}) \quad (2)$$

where:
R = horizontal curve radius of centerline of lane closer to the restrictive element (m), and
M = middle ordinate defined as the horizontal distance between the obstruction element and the centerline of the inside lane (m).

The SSD (i.e., the demand variable) is computed as Equation 3 ,AASHTO (2011).

$$SSD = 0.278V \times PRT + \frac{V^2}{254((\frac{a}{9.81}) \pm g)} \quad (3)$$

where:
V = the operating speed (km/h),
PRT = the perception reaction time (s),
a = the deceleration rate (m/s2), and

g = the longitudinal grade (%).

In this study, all of these variables are probabilistic with many values of SSD for each of the horizontal curves in the dataset and the effect of grade was not considered.

3.2 The second mode accorording to Radius of Horizontal Curve

The limited state funtion(G2) to evaluation the radius of horizontal curves

$$g_2(x) = R_{av} - (\frac{v^2}{127(e+f_s)}) \quad (4)$$

where:
V = the Mean operating speed,
e = the superelevation (i.e. cross slope of the road at horizontal curve segment), and
fs = the demand lateral side friction factor (unit-less).

3. Data Collection Program

3.1 General

The objective of this part is to explain the process that has been followed in the data collection. Therefore, this part contains an explanation of the traffic survey procedure using a digital camera, considered hump in the study, computations used in data processing.

3.2 Traffic Survey Procedure

The traffic was recognized in video films by digital camera. This camera was supported on that tripod in fixed locations. The recorded observation has been carried out along holiday days. Moreover, the vehicles motion was recorded for one as out of peak, from 10.00 am to 11.00 am.

3.3 Considered Horizontal Curves in the Current Study

As the rural freeways highways were targeted by the study, a pilot survey had been carried out for the existing horizontal curves in the scoped roads in Greater Cairo. Results of this survey are listed in Table 1. This survey was aimed to detect the most suitable horizontal curves to study evaluation the safety of available radius (Rav) and available sight distance (ASD). Based on results of this survey, 9 horizontal curves were chosen to be studied lies in Ring highway(no 1 to 5), Alex. Cairo highway(no 6,7) and Al Ain Al Sokhna-Cairo highway (no 8,9).

Table 1: Statistical Summary of Data Set of horizontal curves studied

NO	lanes	Shoulder Width (m)	R (m)	e%	Δ^0	L (m)	Middle Ordinate-M (m)	ASD(m)
1	8	2.50	700	6	105	1283	4.30	156
2	8	2.50	800	5	72	1000	4.30	141
3	8	2.50	1000	4	34	600	4.30	158
4	8	2.50	1250	3.5	57	1240	4.30	177
5	8	2.50	2000	2	60	2070	4.30	222
6	6	12.50	1000	5.6	54	950	14.40	340
7	6	12.50	750	6	77	1000	14.40	292
8	6	4.00	900	4.5	48	750	6.00	208
9	6	4.00	1350	3.5	55	1300	6.00	255

3.4Data Processing

Video tapes were processed using computer, the data had been recorded for each vehicle when it is crossing stations from the beginning to the end of 9 horizontal curves. In particular, the data were collected while processing the video tapes are the time between each two station (PC and PT of curve) in out of peak. The collected data was arranged in spread sheets to be prepared for analysis.

3.5 Computations used in data processing

The collected data from the camera was gathered in Excel sheets and then processed in order to obtain the operating speed. The adopted procedure to get the three parameters is discussed in the following sections.

3.5.1 Operating Speed Calculation (V)

In characterizing the speed of a traffic stream, the space mean speed V_{50} is used. It is the most statistically relevant measure in relationships with other variables, (*HCM2000)*. Space mean speed is computed by dividing the length of the highway section by the average travel time of vehicles traversing it. Travel time (T) of each vehicle is calculated by subtracting the entering time from the exiting time.

3.5.2 Side Friction (Fs)

This study will use data collected by *Himes (2013)* on 15 horizontal curves (16 measurements per curve) as shown table 2. The pavement friction was measured using a Dynamic Friction Tester (DFT), and is controlled mainly by the micro-texture of the pavement surface. The Circular Texture Meter (CTM) was used to measure the pavement macro-texture in terms of mean profile depth (*Himes 2013*).

For the purpose of this research, the values shown in Table 2are interpolated from the results by *Himes (2013)* to correspond to 10 km/h increments using the cubic spline interpolation in Matlab. It should be noted also that the distribution of available lateral friction corresponds to wet pavement conditions, and therefore would correspond only to a specific weather condition.

Table 3 represents the Mean and the Standard Deviation of the Required Lateral friction Each Curve of study

Table 2: Lateral Friction Coefficients for Different Curve Speeds- Reproduced from *Himes (2013)*.

Curve Speed (km/h)		Lateral Friction	
Mean	Standard Deviation	Mean	Standard Deviation
20	2.09	0.6294	0.1112
30	2.92	0.5550	0.0950
40	3.62	0.4885	0.0773
50	4.19	0.4296	0.0640
60	4.61	0.3793	0.0551
70	4.90	0.3341	0.0489
80	5.06	0.2956	0.0461
90	5.07	0.2630	0.0437
100	4.95	0.2310	0.0429
110	4.70	0.2058	0.0431
120	4.31	0.1771	0.0420

Table 3 : the Mean and the Standard Deviation of the Required Lateral Friction for Each Curve

Curve No.	Mean of Operating Speed(km/hr)	Standard Deviation of speed	Mean of fs	Standard Deviation of fs
1	87.79	7.5270	0.2600	0.0237
2	93.27	8.9553	0.2400	0.0265
3	96.24	8.9590	0.2300	0.0255
4	97.69	5.2498	0.2300	0.0149
5	101.44	5.5293	0.2200	0.0150
6	118.66	7.4007	0.1728	0.0162
7	113.74	4.4706	0.1837	0.0106
8	113.39	5.5638	0.1847	0.0132
9	120.98	5.6815	0.1673	0.0122

4. Results and Discussion

4.1 Probability Distributions for the Random Input Parameters

In this study Minitab software is used to calculate the probability of non-compliance by using Monti Carlo simulation method for evaluating the studied horizontal curves. For each curve there is a specific value for the available radius (Rav) and a corresponding value of Pnc. A Monte-Carlo simulation Performed by Minitab was carried out to investing between (Rav) and ($Rdemand$) according to Equation 4.in the other hand , there is a specific value for the ASD and a corresponding value of Pnc. A Monte-Carlo simulation Performed by Minitab was carried out to compare between ASD and SSD according to Equation 1, 2 and 3 Thus, random samples were generated from the probability density functions defined in Table 3.

Table 3 : the Probability Distributions for the Random Input Parameters

Parameter	Mean	Standard Deviation	Distribution	Reference
V	as calculated	as calculated	Normal	
PRT	1.5sec	0.40 sec	Lognormal	Lerner(1995)
a	4.20 m/sec^2	0.60 m/sec^2	Normal	Fambro et al.(1997)
fs	as calculated	as calculated	Normal	*Himes 2013*

4.2 Calculation of Probability of Non-Compliance

Using Minitab program, more than 1000 runs were performed. In each run, the model samples a different value for each random variable and the $Rdemand$ is calculated. There is no strong theoretical support behind the distribution selection for $Rdemand$ for each curve, the distribution selections were instead guided by empirical testing as well as the practicality of implementing the distributions in the Monte-Carlo method.

The empirical tests done for the distribution selection included the Anderson-Darling test and the Pearson Correlation coefficient test. The data were input into Minitab and AD and P-values were determined for all the distributions. A distribution with a relatively lower AD value and a higher P-value indicated a better fitting distribution, given that the P-value is greater than 0.05.The goodness of fit test (i.e., AD and P-value test). A distribution is considered to be the best fit if the data points exactly follow the straight line in as shown in Figure (2) and figure (3). The finalized statistical distributions for available radius and ASD obtained from the analysis in Minitab are shown in Figure(4),(5). The Probability of non-compliance (Pnc) can be calculated by Minitab for curves studied.

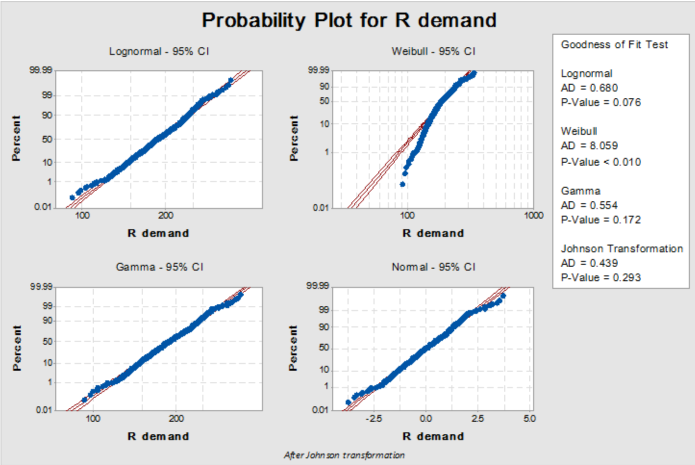

Figure (2): Probability Plot for R demand for Different Probability Distributions Curve NO.1

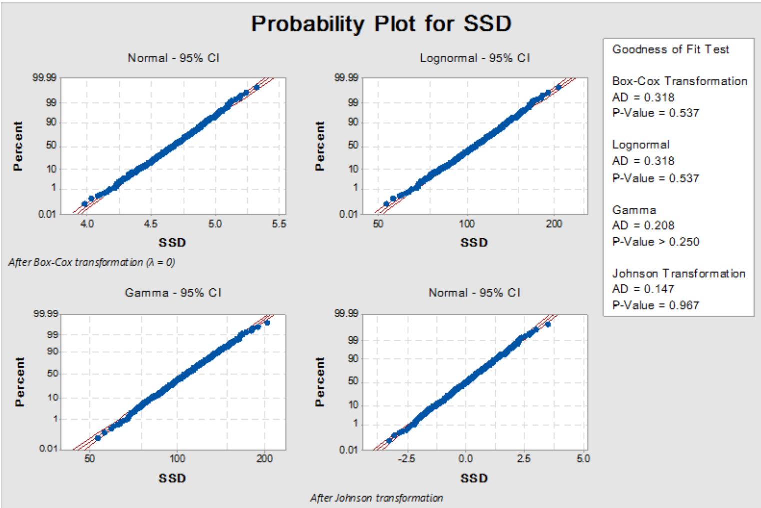

Figure (3): Probability Plot for SSD for Different Probability Distributions Curve No.1

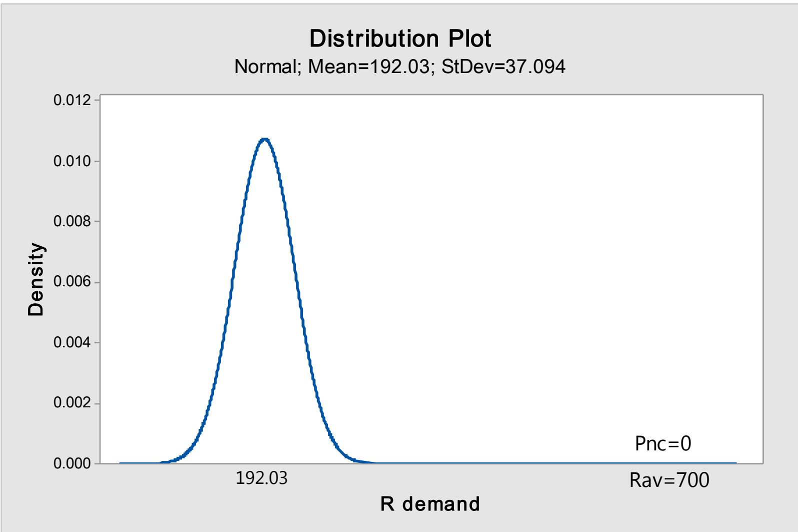

Figure (4): the Probability of Non – Compliance (Pnc) of Rav for Curve NO.1

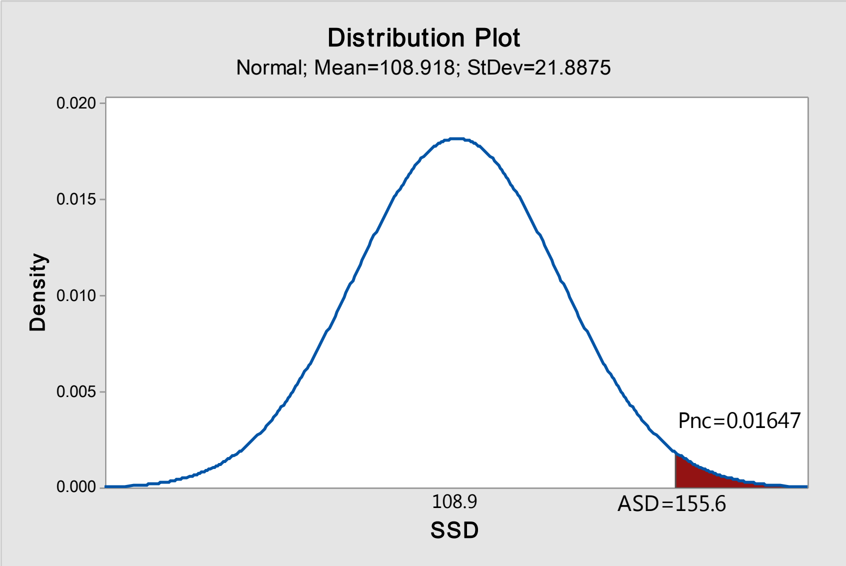

Figure (5): the Probability of Non – Compliance (*Pnc*) of ASD for Curve NO.1

4.3 Results of Probability of Non-Compliance and Reliability Index and Discussion

The table 4 shows values of probability of non-compliance and reliability index as shown eq(6) on two items that 9 horizontal curves in freeways. Overall, it is evident that all The results showed that all curves satisfy the demand radius by users which have a reliability equal 100%.In the other hand, the most curves studied have reliability exceed 90% for the sight distance.

These results occurs because the required side friction is more less than available, that cause the required radius is more than available radius. In the other, the probability of non-compliance of curve 2 is 21.85 % that occur because the middle ordinate M is not large that ASD is not lager enough.

$$R = 1 - Pnc \qquad (6)$$

Table 4 : summary of Reliability Results of Horizontal Curves Elements

Curve No.	Mean of operating speed (km/hr)	R av (m)	ASD (m)	sight distance		Radius	
				Pnc %	R	Pnc %	R
1	87.79	700	156	1.65	98.35	0.00	100
2	93.27	800	141	21.85	78.15	0.00	100
3	96.24	1000	158	12.87	87.13	0.00	100
4	97.69	1250	177	1.76	98.24	0.00	100
5	101.44	1980	222	0.01173	99.99	0.00	100
6	118.66	1000	340	2.234E-05	100	1.6542E-12	100
7	113.74	740	292	0.004797	100	4.1135E-04	100
8	113.39	900	208	7.18	92.82	0	100
9	120.98	1350	255	1.75	98.25	0	100

5. Conclusions

From the analysis of the results, the following conclusions were obtained as follows:

- This study using traffic data collection from sites to calculate the operating speed instead of the previous operating speed prediction models as in the previous works.
- This work was tested using the data from greater Cairo, Egypt. Data were obtained from nine sites. This study considers the design of nine horizontal curves in Cairo, Egypt, where non-compliance occurs whenever the available horizontal curve radius (Rav; supply) falls short of the required radius (R demand). Then, the contributions of uncertainty in the geometric variables were evaluated. The results showed that all curves satisfy the demand radius by users which have a reliability equal 100%.
- Also probability of non-compliance is calculated to evaluate the stopping sight distance , where non-compliance occurs whenever the available sight distance (ASD; supply) falls short of the stopping sight distance (SSD; demand). Then, the contributions of uncertainty in the geometric variables were evaluated. The results showed that about 78% of studied curves have reliability exceed 90%.
- Monte-Carlo simulation was an effective method for implementing the probabilistic analysis approach by using Minitab program. As applied in this case, the Monte-Carlo simulation generated 1000 sets of random input values based on the selected statistical distributions of geometric and traffic characteristics that were developed to obtain a distribution of the SSD and demand radius.

6. Recommendation

The following list contains a number of recommendations for future work:

- The variables in the present study are all considered to be statistically independent, while there is no information that contradicts this assumption; it would be interesting to investigate this and evaluate how the correlation among input variables would affect the outcomes of reliability analysis.
- Current design guidelines can be evaluated to identify the risk associated with each design feature. However, future research could be dedicated to selection of suitable target reliability index (β) and move backwards to find the corresponding design features associated with that risk level. Now we can investigate the relationship between Pnc and collisions, this can be useful for cost benefit analysis and highway safety.
- More required friction models should be considered in future analysis. The current models consider the vehicle to be a single point-mass when traveling within a horizontal curve. The friction demand is currently calculated considering all tires to have the same level of demand, but in reality each tire may have a different demand friction due to the characteristics of the vehicle.
- Finally, more research is needed to study the reliability analysis in different areas of highway design such as structural design of highways and others.

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